Adaptive Set-Point Regulation using Multiple Estimators

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Adaptive Control

Adaptive Control:
A control approach dealing with systems with uncertain or time-varying parameters.
History

Initial results in the 1980s

Original controllers typically **did not** tolerate

- unmodeled dynamics
- time-variation
- noise/disturbances

Attempt to handle these issues

Modify the estimator:

- signal normalization, deadzones, $\sigma$-modification, parameter projection

Attempt to remove or reduce structural assumptions

Pre-routed switching controllers. However, typically yield

- poor transient behavior
- large control input
Multi-Model Adaptive Control

Emergence of non-pre-routed logic-based switching

- Supervisory Control (Morse et al. 1990s): complexity grows with size of plant uncertainty
- Multi-Estimators (Narendra et al. 1990s): shown improvement of transient behavior
- Adaptive Mixing Control (Ioannou & Baldi et al. 2010s): assumes convex uncertainty set
**Typical Properties provided by an Adaptive Controller**

- asymptotic stability
- asymptotic tracking (for minimum phase plants)
- tolerance of slow time-variation and a bit of unmodelled dynamics
- in best case: bounded-noise bounded-output

**What’s not normally proven:**

- bounded gain on the noise
- exponential stability
- any form of convolution bounds
Recent work

Miller 2017 and Miller & Shahab 2018:

For One-Step-Ahead Adaptive Control context and Pole-placement Adaptive Control context:

- exponential stability
- bounded gain on the noise
- linear-like convolution bound

key idea:
- use the original, unmodified, projection algorithm
- restrict parameter estimates to a convex compact set

Extensions

- No convexity assumption in 1st-order one-step ahead adaptive control context (CDC 2018)
- Relaxed convexity assumption and proved stability but not tracking (Miller & Shahab 2018)
In this paper

Objectives
- Remove **convexity** requirement
- Step tracking (Set-point control)

Our Approach
- Estimate parameters of an **auxiliary** plant rather than the original one
- Use multiple estimators
- Pole-placement-based switching control law
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Plant

\[ y(t + 1) = \sum_{j=1}^{n} a_j y(t - j + 1) + \sum_{j=1}^{n} b_j u(t - j + 1) + w(t), \quad t \in \mathbb{Z} \]

- Plant parameters \( \theta = [a_1 \ a_2 \ \cdots \ a_n \ b_1 \ b_2 \ \cdots \ b_n]^\top \in S \subset \mathbb{R}^{2n} \)
- Associated with this plant model are the polynomials
  \[ A(z^{-1}) = 1 - a_1 z^{-1} - \cdots - a_n z^{-n}, \quad B(z^{-1}) = b_1 z^{-1} + \cdots + b_n z^{-n} \]

Assumption 1

- \( S \) is compact (closed and bounded)
- \( A(z^{-1}) \) and \( B(z^{-1}) \) are coprime

Assumption 2 (Step Tracking)

- \( B(1) \neq 0 \)
The Auxiliary Plant

- Classical approach: estimate plant parameters and force integrator into the corresponding pole-placement based control law
- $S$ is compact not convex $\Rightarrow$ union of convex sets; multiple plant parameter estimators $\Rightarrow$ prove stability but not tracking
- However, to **prove** the desired linear-like behavior and tracking we propose:

**System identification done on a related auxiliary model**

- For set-point $y^*$, we can write the auxiliary plant:

  $$\bar{y}(t + 1) = \bar{\phi}(t)^\top \theta^* + \bar{w}(t)$$

- Tracking error: $\bar{y}(t) = y(t) - y^*$
- Auxiliary input: $\bar{u}(t) = u(t) - u(t - 1)$
- Parameters $\theta^* = [\bar{a}_1 \ldots \bar{a}_{n+1} b_1 \ldots b_n]^\top \in \bar{S} \subset \mathbb{R}^{2n+1}$
- $\bar{\phi}(t) := [\bar{y}(t) \ldots \bar{y}(t - n) \bar{u}(t) \ldots \bar{u}(t - n + 1)]^\top$
The Auxiliary Plant

- Classical approach: estimate plant parameters and force integrator into the corresponding pole-placement based control law
- $S$ is compact not convex $\Rightarrow$ union of convex sets; multiple plant parameter estimators $\Rightarrow$ prove stability but not tracking
- However, to **prove** the desired linear-like behavior and tracking we propose:

**System identification done on a related auxiliary model**

Auxiliary parameters are determined in a simple way from those of $A(z^{-1})$:

\[
\bar{A}(z^{-1}) = (1 - z^{-1})A(z^{-1}) = 1 - (1 + a_1)z^{-1} - (a_2 - a_1)z^{-2} - \cdots
\]

\[
=:\bar{a}_1 \quad =:\bar{a}_2
\]

\[
\cdots - (a_n - a_{n-1})z^{-n} - (-a_n)z^{-(n+1)}
\]

\[
=:\bar{a}_n \quad =:\bar{a}_{n+1}
\]

The set of admissible parameters of the auxiliary plant:

\[
\bar{S} := \left\{ \mathbf{V} \begin{bmatrix} 1 \\ \theta \end{bmatrix} : \theta \in S \right\}
\]
Uncertainty Sets

Proposition

- There exist a finite number $m$ of **compact** and **convex** sets $S_i \subset \mathbb{R}^{2n+1}$ that satisfy
  \[ \bar{S} \subset \bigcup_{i=1}^{m} S_i \]

- Each $S_i$ satisfy **Assumption 1** and **Assumption 2**

Note: $i^* := \min \{ i \in \{1, 2, \ldots, m\} : \theta^* \in S_i \}$
Modified Projection Algorithm for $S_i$:

- **Prediction error**
  \[ e_i(t + 1) = \bar{y}(t + 1) - \phi(t)^\top \hat{\theta}_i(t) \]

- **Turning-off mechanism** (with $\bar{s} := \max_i \|S_i\|$)
  \[ \rho_i(t) := \begin{cases} 1 & \text{if } |e_i(t + 1)| < (2\bar{s} + \delta)\|\phi(t)\| \\ 0 & \text{otherwise} \end{cases} \]

- **Estimator Update**
  \[ \tilde{\theta}_i(t + 1) = \hat{\theta}_i(t) + \rho_i(t) \times \frac{\phi(t)}{\|\phi(t)\|^2} e_i(t + 1) \]
  \[ \hat{\theta}_i(t + 1) = \text{Proj}_{S_i} \{ \tilde{\theta}_i(t + 1) \} . \]
Adaptive Controller: Control Design

Pole-placement based design:
For each $i \in \{1, 2, \ldots, m\}$,

- With $\hat{\theta}_i(t) =: [\hat{a}_{i,1}(t) \ \cdots \ \hat{a}_{i,n+1}(t) \ \hat{b}_{i,1}(t) \ \cdots \ \hat{b}_{i,n}(t)]^\top$:

$$\begin{align*}
\hat{A}_i(t, z^{-1}) &= 1 - \hat{a}_{i,1}(t)z^{-1} - \cdots - \hat{a}_{i,n+1}(t)z^{-(n+1)} \\
\hat{B}_i(t, z^{-1}) &= \hat{b}_{i,1}(t)z^{-1} + \cdots + \hat{b}_{i,n}(t)z^{-n}.
\end{align*}$$

- Design a $(n + 1)^{th}$-order strictly proper controller: choose

$$\begin{align*}
\hat{L}_i(t, z^{-1}) &= 1 + \hat{l}_{i,1}(t)z^{-1} + \cdots + \hat{l}_{i,n}(t)z^{-n} \\
\hat{P}_i(t, z^{-1}) &= \hat{p}_{i,1}(t)z^{-1} + \cdots + \hat{p}_{i,n+1}(t)z^{-(n+1)}
\end{align*}$$

- To place all closed-loop poles at the origin:

$$\begin{align*}
\hat{A}_i(t, z^{-1})\hat{L}_i(t, z^{-1}) + \hat{B}_i(t, z^{-1})\hat{P}_i(t, z^{-1}) &= 1.
\end{align*}$$
Adaptive Controller: Switching Control Law

- Control gains:
  \[ \hat{K}_i(t) := [-\hat{p}_{i,1}(t) \cdots -\hat{p}_{i,n+1}(t) -\hat{l}_{i,1}(t) \cdots -\hat{l}_{i,n}(t)] \]

- Control law:
  \[ \bar{u}(t) = \hat{K}_{\sigma(t-1)}(t-1)\bar{\phi}(t-1) \]
  \[ u(t) = \bar{u}(t) + u(t-1) \]

- Switching signal: \( \sigma(\cdot) \in \{1, 2, \ldots, m\} \) piecewise constant:
  \[ \sigma(t) = \sigma(\hat{t}_\ell), \quad t \in [\hat{t}_\ell, \hat{t}_{\ell+1}) \]
  with switching times
  \[ \hat{t}_\ell = t_0 + \ell \times N, \quad \ell = 0, 1, 2, \ldots \]
Adaptive Controller: Switching Algorithm

Performance signal: \[ J_i(\hat{t}_\ell) = \sum_{j=\hat{t}_\ell}^{\hat{t}_{\ell+1}-1} \rho_i(j) \times \frac{|e_i(j+1)|}{\|\overline{\phi}(j)\|} \]

Switching Algorithm: with \( \mathcal{I}(\hat{t}_0) = \{1, 2, \ldots, m\} =: \mathcal{I}^* \),

\[
\hat{\mathcal{I}}(\hat{t}_\ell) = \left\{ i \in \mathcal{I}^* : J_i(\hat{t}_\ell) < J_{\sigma(\hat{t}_\ell)}(\hat{t}_\ell) \right\}
\]

\[
\mathcal{I}(\hat{t}_{\ell+1}) = \begin{cases} 
\mathcal{I}^* & \text{if } \mathcal{I}(\hat{t}_\ell) \cap \hat{\mathcal{I}}(\hat{t}_\ell) = \emptyset \\
\mathcal{I}(\hat{t}_\ell) \cap \hat{\mathcal{I}}(\hat{t}_\ell) & \text{otherwise} 
\end{cases} \quad (\text{Index set reset})
\]

\[
\sigma(\hat{t}_{\ell+1}) = \arg\min_{i\in\mathcal{I}(\hat{t}_{\ell+1})} J_i(\hat{t}_\ell)
\]

Lemma

For every \( t_0 \in \mathbb{Z}, y^* \in \mathbb{R}, \sigma(\hat{t}_0) \in \mathcal{I}^*, \overline{\phi}(t_0) \in \mathbb{R}^{2n+1}, \theta^* \in \tilde{S}, N \geq 1, \hat{\theta}_i(t_0) \in \mathcal{S}_i \ (i \in \mathcal{I}^*), \) and \( w \in \ell_\infty, \) if \( \hat{t}_\ell \) and \( \hat{t}_{\ell} \) are two consecutive reset times of the index set, then there exists a \( \ell^* \in [\ell, \bar{\ell}) \) such that:

\[ J_{\sigma(\hat{t}_{\ell^*})}(\hat{t}_{\ell^*}) \leq J_{i^*}(\hat{t}_{\ell^*}). \]
Main Result

With \( \phi(t) = [y(t) \; \cdots \; y(t-n+1) \; u(t) \; \cdots \; u(t-n+1)]^\top \)

**Theorem**

For every \( \lambda \in (0, 1) \), \( \delta \in (0, \infty] \) and \( N \geq 2n + 1 \), there exists a constant \( \gamma > 0 \) such that for every \( t_0 \in \mathbb{Z} \), \( \phi_0 \in \mathbb{R}^{2n+2} \), \( \sigma(\hat{t}_0) \in \mathcal{I}^* \), \( \theta \in S \), \( \hat{\theta}_i(t_0) \in S_i \) (\( i \in \mathcal{I}^* \)), \( y^* \in \mathbb{R} \) and \( w \in \ell_{\infty} \), the following holds

\[
\| \phi(t) \| \leq \gamma \lambda^{t-t_0} \| \phi_0 \| + \gamma |y^*| + \gamma \sum_{j=t_0}^{t-1} \lambda^{t-1-j} |w(j)|
\]

If \( w \) is constant: \( y(t) \to y^* \)

- Linear-like closed-loop behavior with
  - uniform exponential bound w.r.t. any initial condition
  - convolution sum bound w.r.t. exogenous signals
  - bounded gain on the noise

- Step Tracking when constant noise/disturbance

- The approach does not assume the switching to stop

- The approach is modular: we can use the above theorem to prove tolerance to time-variations & unmodelled dynamics
**Simulation Example**

Consider the 2\textsuperscript{nd}-order plant:

\[ y(t + 1) = a_1 y(t) + a_2 y(t - 1) + b_1 u(t) + b_2 u(t - 1) + w(t) \]

with parameter uncertainty set

\[ S := \left\{ \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix}^\top \in \mathbb{R}^4 : a_1 \in [-2, 0], a_2 \in [-3, -1], b_1 \in [-1, 0], b_2 \in [-5, -3] \cup [3, 5] \right\} \]

\( S \) is not convex and its convex hull includes the case \( b_1 = b_2 = 0 \)

- We define the set \( \bar{S} \) to estimate parameters of the auxiliary plant
- Then we define

\[ S_1 := \left\{ \begin{bmatrix} \bar{a}_1 & \bar{a}_2 & \bar{a}_3 & b_1 & b_2 \end{bmatrix}^\top \in \mathbb{R}^5 : \bar{a}_1 \in [-1, 1], \bar{a}_2 \in [-3, 1], \bar{a}_3 \in [1, 3], b_1 \in [-1, 0], b_2 \in [-5, -3] \right\} \]

\[ S_2 := \left\{ \begin{bmatrix} \bar{a}_1 & \bar{a}_2 & \bar{a}_3 & b_1 & b_2 \end{bmatrix}^\top \in \mathbb{R}^5 : \bar{a}_1 \in [-1, 1], \bar{a}_2 \in [-3, 1], \bar{a}_3 \in [1, 3], b_1 \in [-1, 0], b_2 \in [3, 5] \right\} \]

\( \bar{S} \subset S_1 \cup S_2 \) and satisfies required assumptions
Simulation Example

- Unknown plant parameters: \( a_1 = -\frac{1}{2}, a_2 = -\frac{3}{2}, b_1 = -\frac{3}{4}, b_2 = -3 \)
- \( N = 5, \delta = \infty \)
- Set-point: \( y^* = 2 \)
- \( |w(t)| = \frac{1}{2} \) but its sign changing every 250 steps
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Summary

- Uncertainty set only assumed to be **compact**
- ... and satisfying coprimeness assumptions

**Adaptive Controller:**
- estimate parameters of auxiliary model
- multiple projection-algorithm based estimators
- switching pole-placement based control law

**Linear-like** closed-loop behavior with
- uniform exponential bound
- bounded gain on the noise
- convolution sum bound

**Set-point Tracking**
Future Work

What’s Next?

- Extend to cases when the plant order is unknown
- Extend to track more general reference signals
- Study further performance of the transient behavior
Thanks

- Thanks to the reviewers
- Thanks to session chairs
- Thanks to 2019 IEEE CDC organizing committee

Q&A

Happy Holidays !!